AN ATTEMPT TO SCALE REYNOLDS STRESSES IN PRESSURE GRADIENT TURBULENT BOUNDARY LAYER

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1 Introduction

Recent studies deal with the scaling of Reynolds stresses for zero pressure gradient flows and for high Reynolds number. The particular attention is given to the appearance of a second, so-called outer maximum of the streamwise $\overline{uu}$ Reynolds stresses (Marusic et al. 2010). Quite recently Harun et al. (2013) have demonstrated the appearance of outer peak in adverse pressure gradient (APG) flows and concluded that the constant scaling factor throughout the boundary layer thickness is not appropriate for Reynolds stress profiles. It seems that remedy for this problem could be the proposal of Alfredsson et al. (2012) developed for wall-bounded turbulent flows, where the scale is variable and equal to local mean velocity U. They show that the streamwise turbulence intensity u'/U in the outer region appears to collapse on a straight line with a functional dependence on U/U_∞. The parameters of that line for turbulent boundary layers are described by following equation:

$$\frac{u'}{U} = 0.286 - 0.255\frac{U}{U_\infty}. \qquad (1)$$

To obtain the collapse of data also in the near wall region Alfredsson et al. (2012) proposed the difference function:

$$\Delta(U^+) = \frac{u'}{U} - 0.286 - 0.255\frac{U}{U_\infty}. \qquad (2)$$

where $U^+ = U/u_\tau$, $u_\tau = (\tau_w/\rho)^{1/2}$ is friction velocity, τ_w is wall shear stress and ρ is fluid density.

The authors (Alfredsson et al. 2012) show that the scaling is valid for turbulent boundary layers in zero pressure gradient, at least for analysed cases in the paper.

The paper presents the attempt to modify the scaling proposed by Alfredsson et al. (2012) in order to obtain collapse in pressure gradient flows. The data used in the analysis come from the experiment performed for the pressure gradient conditions representative for practical turbomachinery flows, where sudden change from favourable (FPG) to APG occurs (Dróżdż and Elsner 2013).

2 Experimental data

Detailed analysis was performed based upon 24 profiles (see Figure 1) measured with single hot-wire probe. Reynolds number were varying from Re_θ = 2300 ÷ 6200. The skin friction u_τ values were obtained based on Fringe Skin Friction (FSF) method and Clauser plot. The mean velocity profiles measured in consecutive measuring traverses allowed to determine the evolution of turbulent boundary layer along the flat plate. The values of u_τ and H show that the turbulent boundary layer has not yet separated.

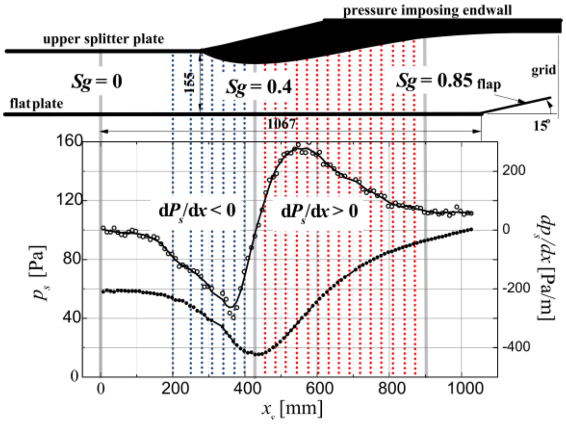

Figure 1: The shape of channel upper wall with corresponding static pressure P_s and pressure gradient dP_s/dt distributions.

The sample experimental data (see Figure 2) show that turbulent boundary layer reacts differently to FPG and to APG conditions, the FPG causes the decrease, while APG causes strong increase of streamwise Reynolds stress in the outer region.

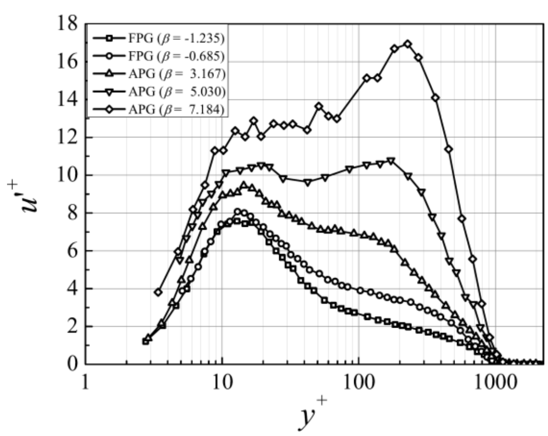

Figure 2: Streamwise Reynolds stress profiles scaled on u_τ.

3 Streamwise Reynolds stress scaling

In order to scale properly u' in pressure gradient flows, the additional weighting factor should to be considered. One of the scaling factors candidate is the one proposed by Zagarola & Smits (1998) δ^*/δ, however, as it turned out from initial analysis (not shown here) it did not work properly, because after it's application profiles were over-scaled and lie below the linear fit. Additionally, δ^*/δ could not be treated as a proper scaling factor, because it often has constant value for non-zero pressure gradient, especially for high Reynolds numbers. Instead, in the paper we propose to use the shape factor $H = \delta^*/\theta$.

The use of the additional scaling by shape factor seems to be valid for boundary layers with pressure gradient because of characteristic feature of H, namely its weak dependence on Reynolds number and strong dependence on pressure gradient.

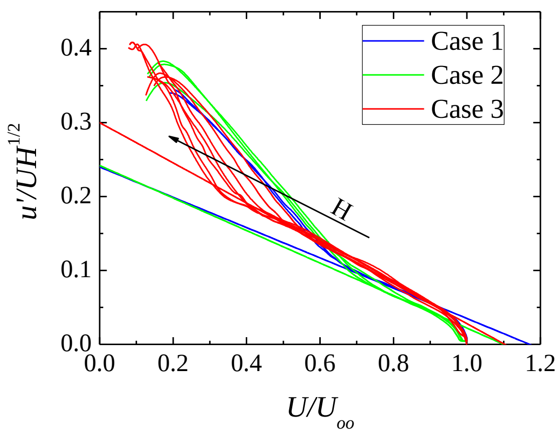

Figure 3: Streamwise Reynolds stress ($\overline{uu}/(U^2H))^{1/2}$ scaled with modified AOS scaling.

The profiles of $(\overline{uu}/(U^2H))^{1/2}$ are presented in Figure 3. As can be seen its application ended up with partial success, namely the convergence was obtained into three different linesof various inclination. After further analysis it seems that these three lines correspond to three different flow states (see Table 1). The boundaries among these states are defined by locations of distinct minimum or maximum of pressure gradient (see Figure 1).

Table 1: Parameters of the linear lines

Case:	Conditions of PG:	B	A
1	ZPG following FPG $d^2p/dx^2 < 0$	-0.205	0.24
2	FPG following APG $d^2p/dx^2 > 0$	-0.22	0.24
3	FPG following APG $d^2p/dx^2 < 0$	-0.27	0.30

We propose the complete difference function for streamwise Reynolds stresses written in the following form:

$$\Delta_H(U^+H) = \frac{\overline{uu}}{U^2H} - \left(A + B\frac{U}{U_\infty}\right)^2, \qquad (4)$$

where A and B depends upon the sequence of upstream pressure gradient conditions (see Table 1). Figure 4 shows the difference function (Eq. 4) for analysed data. Profiles are grouped in five bundles, (marked as different colour in Figure 4). In the group the profiles collapse all across boundary layer thickness.

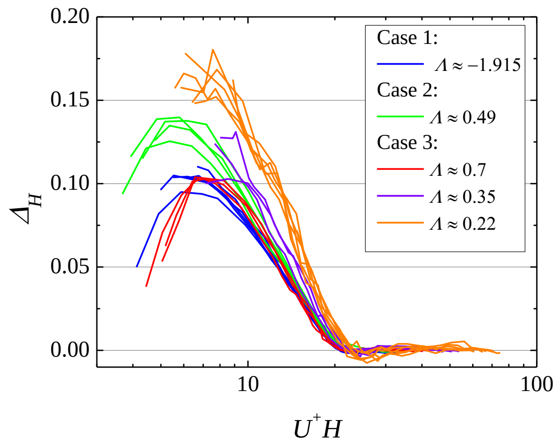

Figure 4: Complete difference function (Eq. 4).

What is interesting the collapse occurs for equilibrium criterion introduced by Castillo-George i.e. constant pressure gradient parameter Λ:

$$\Lambda = \frac{\delta}{\rho U_\infty^2 d\delta/dx}\cdot\frac{dp_\infty}{dx}, \qquad (3)$$

where δ is boundary layer thickness. In Figure 5 linear distributions of constant Λ in the FPG and APG regions can be observed, which means that analysed boundary layer is a non-equilibrium one, but it remains in local equilibrium.

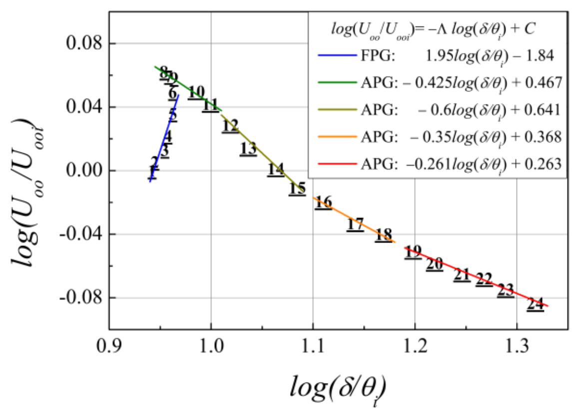

Figure 5: Pressure gradient parameter Λ: distribution of $\log(U_\infty/U_{\infty i})$ versus $\log(\delta/\theta_i)$.

The collapse occurs because the self-similar profiles of velocity deficit were obtained (Drobniak et al. 2011) for analyzed flow when they are scaled by Zagarola-Smits scaling (Zagarola & Smits 1998). It was

suspected that when local mean velocity is applied to Equation 4 it has to reflect the self similarity of mean velocity profiles. Furthermore, it can be observed (Figure 5) that if the pressure gradient parameter Λ decreases the maximum of the complete difference function increases. Thus, if complete difference function is modified in following way:

$$\Delta_H \Lambda^{n/2} = \frac{\overline{uu}}{U^2 H} - \left(A + B \frac{U}{U_\infty} \right)^2 , \qquad (5)$$

where n is the sign of Λ, the profiles collapses all together, what is shown in Figure 6. The differences are visible only very close to the wall, in viscous sublayer due to thermal effect of the wall on hot-wire probe.

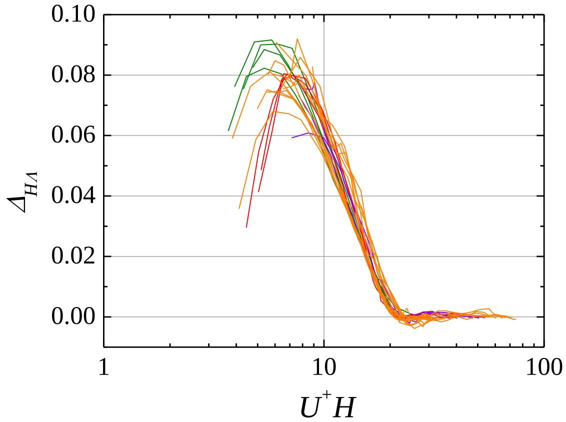

Figure 6: Self-similar complete difference function.

The streamwise Reynolds stresses of analysed turbulent boundary layer in pressure gradient conditions can be calculated by following formula:

$$\overline{uu} = \left(\Delta_H \Lambda^{n/2} + \left(A + B \frac{U}{U_\infty} \right)^2 \right) U^2 H . \qquad (6)$$

4 Conclusion

The substantial change of fluctuation distributions, which may be attributed to a complexity of the analysed case, is the reason for the lack of the self-similarity of Reynolds stress profiles. At given conditions the self-similarity cannot be obtained using the scale, which is constant across the boundary layer thickness. The analysed flow is characterised by strong APG region preceded by strong FPG region, which results in few local equilibrium regions defined by constant pressure gradient parameter Λ.

The new proposal of streamwise Reynolds stresses scaling, which is based on Alfredsson et al. (2012) concept was introduced. It extends the applicability of the letter one on pressure gradient turbulent boundary layers by means of additional scaling factor, which is the product of U^2 and shape factor H. This expression takes into account the change of mean velocity profile and corrects the streamwise Reynolds stress in the outer region, which is especially important for APG conditions. Pressure gradient parameter Λ corrects the complete difference profiles close the wall so the profiles collapse all together across boundary layer thickness.

Acknowledgements

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